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*SEP Thrust Subsystem Performance  
Sensitivity Analysis*

*K. L. Atkins*

*C. G. Sauer, Jr.*

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JET PROPULSION LABORATORY  
CALIFORNIA INSTITUTE OF TECHNOLOGY  
PASADENA, CALIFORNIA

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## PREFACE

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## ABSTRACT

This is a two-part report on solar electric propulsion (SEP) performance sensitivity analysis. The first part describes the preliminary analysis of the SEP thrust system performance for an Encke rendezvous mission. A detailed description of thrust subsystem hardware tolerances on mission performance is included together with nominal spacecraft parameters based on these tolerances.

The second part of this report describes the method of analysis and graphical techniques used in generating the data for Part 1. Included is a description of both the trajectory program used and the additional software developed for this analysis. Part 2 also includes a comprehensive description of the use of the graphical techniques employed in this performance analysis.

## PART 1. HARDWARE TOLERANCES AND TRADEOFFS

### I. INTRODUCTION

#### A. Hardware Description

A low-thrust trajectory is a radical departure from the traditional ballistic trajectory in propulsion system operations. The SEP hardware will be in operation for months rather than minutes, and trajectory energy is imparted in continuous fashion over a significant time period. Achievement of the desired final state vector, however, is still dependent on the ability to predict and control the hardware performance over the increased operation time.

Because the propellant is expended on an atomic scale, control must be exercised in an indirect manner. For instance, because no direct measurement of mass flow rate is available, the flow rate must be controlled from some a priori calibration. Present control schemes utilize the relationship between the discharge power and the mass utilization efficiency, as indicated by the ion beam current, to regulate propellant flow. Calibrations are made for the individual thrusters, and these calibrations are assumed to be accurate in flight. The difficulty with this scheme is in the sensitivity of the calibration to a number of thruster parameters, including thruster geometry, magnetic field strength and shape, division of flow between main and cathode vaporizers, etc. These calibrations will also vary in time as a function of component aging, line and load variations, and subsystem random perturbations.

The effective specific impulse  $I_s$  and overall efficiency  $\eta_{TSS}$  of a SEP thrust subsystem are subject not only to calibration uncertainties but to variations with input power and time. The effects of these uncertainties and variations on the trajectory must be understood and the knowledge used to set hardware limits which ensure mission success. These limits must be the development standards for thrust subsystem hardware.

The  $I_s$  and  $\eta_{TSS}$  are related to thrust subsystem parameters as follows:

$$I_s = \frac{1}{mg} \sqrt{\frac{2e}{M}} \sum_{i=1}^N \dot{m}_i \left( \eta_{1_i} + \sqrt{2} \eta_{2_i} \right) \sqrt{V_{B_i}} \overline{\cos \theta_i} \cos \alpha_i \cos \beta_i \zeta_i \quad (1)$$

$$\eta_{TSS} = \frac{1}{N} \sum_{i,j=1}^N \eta_{c_j} \eta_{PC_j} \frac{\left( \eta_{1_i} + \sqrt{2} \eta_{2_i} \right)^2 V_{B_i} I_{B_i}}{\left( \eta_{1_i} + 2\eta_{2_i} \right)^2 P_{TH_i}} \times \overline{\cos \theta_i}^2 \cos^2 \alpha_i \cos^2 \beta_i \zeta_i^2 \quad (2)$$

The summations are carried out over each operating thruster  $i$  and each power conditioner  $j$ . The various terms are defined as follows:

- $F$  = thrust level delivered by the subsystem, N
- $\dot{m}$  = propellant mass flowrate, kg/s
- $g$  =  $9.78 \text{ m/s}^2$
- $e$  = electronic charge =  $1.6 \times 10^{-19}$
- $M$  = mass of the propellant atom =  $3.34 \times 10^{-25}$  kg (mercury)
- $\eta_1$  = fraction of  $\dot{m}$  exiting as singly ionized mercury atoms
- $\eta_2$  = fraction of  $\dot{m}$  exiting as doubly ionized mercury atoms
- $V_B$  = net potential through which ions are accelerated
- $I_B$  = ion current in the exhaust beam =  $(\eta_1 + 2\eta_2) \frac{e}{M} \dot{m}$
- $P$  = power available to the thrust subsystem for conversion to thrust
- $\eta_c$  = cabling efficiency
- $\eta_{PC}$  = power conditioner efficiency
- $N$  = number of operating thrusters
- $P_{TH}$  = power available to an individual thruster  $\approx \eta_c \eta_{PC} \frac{P}{N}$

$\overline{\cos \theta}$  = beam divergence factor

$\alpha$  = gimbal angle

$\beta$  = thrust vector misalignment angle

$\zeta$  = factor for thrust recovery from charge exchange, deposition,  
.....  
etc.

In the ideal case, each of these parameters would be held rigidly constant with the exception of  $P$ ,  $P_{TH}$ ,  $m$  and  $I_B$ , and the latter three would vary in a known and predictable manner with  $P$ . In practice, none of these parameters are constant.

Through Eqs. (1) and (2) the individual parameter uncertainties are combined. The combined parameters,  $I_s$  and  $\eta_{TSS}$ , directly enter the equations of motion and characterize the subsystem for trajectory performance. Thus, examination of the effects of variances in these combined parameters on the trajectory and a subsequent setting of acceptable variance limits are the first step in defining specifications for the individual subsystem parameters.

#### B. Mission Interfaces

The magnitude of the instantaneous thrust acceleration supplied by the SEP system is related to the parameters  $I_s$  and  $\eta_{TSS}$  by

$$a = \frac{2\eta_{TSS}P}{mI_s g} \quad (3)$$

where  $m$  is the instantaneous mass being accelerated. The acceleration couples the system hardware parameters to the trajectory performance through the equation of motion,

$$\ddot{\underline{r}} + K \frac{\underline{r}}{r^3} = \underline{a} = \frac{2\eta_{TSS}P(r)}{m I_{sp} g} \underline{u} \quad (4)$$

where  $\underline{r}$  is the position vector,  $K$  represents the gravitational constant, and  $\underline{u}$  is the unit vector of the applied acceleration;  $P(r)$  gives the available power from the solar arrays as a function of position.

A successful trajectory has three important constituents: (1) reaching the desired position and velocity, (2) with the required amount of hardware, and (3) within a specified interval of time. Prediction of success is achieved when the equation of motion has been integrated over the trajectory to reach the desired final position and velocity. A determination of mass is implied but it is an additional unknown in Eq. (4). The relationship for the mass flow rate in terms of the system parameters is obtained by

$$\dot{m} = \frac{2\eta_{TSS}P(r)}{I_s^2 g^2} \quad (5)$$

Equations (4) and (5) thus allow the study of trajectory sensitivity to the combined hardware parameters  $I_s$  and  $\eta_{TSS}$ .

## II. SUMMARY OF RESULTS

A 1261-kg spacecraft with 20 kW of installed power at 1 AU and 16 kW delivered to the thrust subsystem was considered for a 950-day Encke rendezvous mission launched in 1978 and arriving at Encke in 1980. The thrust subsystem nominal  $I_s$  was 3000 s and the efficiency was 65%. For this case, the hardware constraints were a minimum delivered  $I_s$  of 2910 s and a minimum efficiency of 61.5%. To provide for these tolerances, the propellant reserve would have to be 56 kg.

The nominal subsystem efficiency was based on the assumption that the parameters  $\eta_1$ ,  $\eta_2$ ,  $\overline{\cos \theta}$ ,  $\cos \alpha$ ,  $\cos \beta$ , and  $\zeta$  in Eqs. (1) and (2) have values of  $\eta_1 = 0.9$ ,  $\eta_2 = 0$ ,  $\overline{\cos \theta} = \cos \alpha = \cos \beta = \zeta = 1$ . As these parameters are varied, both specific impulse and efficiency vary. Variations of  $\overline{\theta}$  and  $\eta_2$  were specifically examined to determine limits caused by the trajectory. The results indicated that only small variations in these parameters could be tolerated for fixed flight time trajectories. For example, for  $\overline{\theta} = 5$  deg, the maximum allowable value of  $\eta_2$  is about 0.035.

The study also showed that constraints on the hardware performance could be relaxed by the addition of more power. Increasing the power level of the thrust subsystem by 1 kW, for example, drops the minimum acceptable efficiency at 3000 s to 60.2%, and at 2900 s to 58.6%. Thus, the power

level significantly influences hardware constraints. Since power, however, is a major cost item, there could be a strong motivation to hold to the lowest permissible power level. To do this requires (1) a good knowledge of the actual performance of the thrust subsystem at the time the power level is selected, and (2) tight constraints thereafter to meet that performance.

### III. CONCLUSIONS

The major conclusions reached in the study are:

- (1) Uncertainties in achievable thrust-subsystem performance must be considered in selecting both the power level of the spacecraft and the ion-beam voltage.
- (2) Any reasonable variance in the thrust subsystem performance can be accommodated by increasing the power level.
- (3) Once the power level and beam voltage have been selected, hard limits are set on thrust subsystem performance. Violation of these limits will make the mission unattainable with the original constraint of fixed flight time.
- (4) On the basis of the above, an accurate knowledge of true subsystem performance is essential prior to the final selection of a design power level and beam voltage; otherwise, the final selection of the power level must be based on worst-case assumptions of thrust subsystem performance.

### IV. APPROACH

#### A. Study Guidelines

The equations of motion are subject to the hardware controls available in the thrust acceleration term. In addition to the combined subsystem parameters under investigation  $I_s$  and  $\eta_{TSS}$ , the controls include the amount of time the system is operated, the initial mass which must be accelerated, and the time history of the thrust pointing vector  $\underline{u}$ . To perform a detailed or total study of trajectory sensitivity to the hardware parameters, variances in the additional controls should be included. Each control should be



optimized in the sense that histories (e. g., thrust-coast times and pointing vector) which ensure a successful trajectory but which, at the same time, place the least restriction on the thrust subsystem operating specifications, would be selected. The objective should be to determine the set of paths over a desired launch opportunity which exhibit these features:

- (1) A relatively low amount of thrust time per thruster, thereby increasing reliability through a reduction of hardware operation time.
- (2) Placement of coasts, which could be used as thrust periods to increase trajectory tolerance to substandard hardware performance.
- (3) A thrust pointing history minimizing the number of vehicle inertial attitude changes.
- (4) Trajectory success over a wide range of  $I_s$  and  $\eta_{TSS}$ .

The above features are parametric constraints in the mathematical formulation to determine these paths. Because the equations of motion are nonlinear and because of the possible discontinuous nature of controls, simulation and study of the constraints are difficult and time-consuming, even on the fastest computers. Bounding the controls significantly increases the complexity and overconstrains the problem. Current trajectory analysis programs have therefore been formulated as optimizers of the final mass with freedom from constraints, which allows adequate performance analysis with increased computational speed, while keeping analyses costs relatively low. As a result, only limited capability exists for any detailed simulation of desired trajectory and spacecraft constraints. For these reasons, the results of this analysis study are classed as preliminary in the sense that they apply only to the simplified model used in this study.

The study guideline enforced by the analysis tools and the available study time was to determine the set of paths exhibiting feature (4) under the constraint of feature (1). Thus, trajectories were required to have coast phases, but accurate quantitative thrust times were not determined. Further, it was not feasible to examine the effects of thrust-period placement or constrained thrust angles on the tolerances for the collective parameters  $I_s$  and  $\eta_{TSS}$ . Note that the omission of features (2) and (3) leaves the probability

of possible future changes in the acceptable hardware performance limits. The importance of early tolerance specifications for hardware development raises the priority for securing fast, accurate, flexible, and inexpensive hardware simulation programs to alleviate the guideline restrictions of this study.

## B. Trajectory Analysis

A large number of trajectories displaying the effects of hardware parameter changes must be studied to determine how much variation the trajectory can tolerate without jeopardizing mission success. The basic data can then be analyzed with selected mission success standards, and the boundaries for the hardware parameters can then be determined. A recently developed optimization program (Ref. 1), which features fast trajectory computations through approximation techniques, was employed to perform a sensitivity analysis of a 1980 Encke rendezvous mission. Trajectories were determined for a range of launch dates consistent with this mission opportunity. Because of the large number of interacting hardware and mission parameters, the initial approach was to reduce the total number of parameters to be considered in detail. A range of launch energies was selected. All masses were normalized and trajectories generated for a spectrum of launch energies. Parameters were mission time and  $I_s$ . The initial acceleration  $a_0$  for each path was examined as a function of the launch-energy spectrum, as shown in Fig. 1. Initial acceleration, in a heuristic sense, is indicative of the amount of energy to be supplied by the thrust subsystem. As launch speed increases, the required initial acceleration decreases up to a certain point. Over the range above 4 km/s the amount of launch excess speed has little effect. A speed typical of this range (8 km/s) was selected for use in detailed analysis, thus reducing the energy parameters under consideration to a single representative value in a manner independent of launch vehicle capabilities.

The mission times were also quickly reduced, as shown in Fig. 2. At times below about 950 days the selected optimization quantity, the ratio of final mass to initial jet power, rapidly decreases. The reasons for selection of ratios like this as optimization functions have been discussed in the open literature (Ref. 2), and the reasons for the particular choice made during

the study are discussed in section V-A. An allowance was made for auxiliary spacecraft power  $\Delta P$  through inclusion of a  $\Delta P/P_j$  of 0.02 divided by efficiency.

After the parameters were reduced, the study could be focussed on the generation of detailed data. Before this could take place, however, mission success boundaries had to be defined.

### C. Trajectory Success Boundaries

1. Hardware and science considerations. The acceptable final state of the vehicle is strongly influenced by the science objectives. Because of the emphasis on hardware technology, the science role was minimized and authoritative science boundaries for the mission were not established; however, some arbitrary assumptions were made. For example, the closer to perihelion that rendezvous occurs, the larger the masses that can be delivered by a given thrust subsystem. Thus, from a hardware standpoint, it is desirable to arrive as close to perihelion as possible. However, arrivals close to perihelion may leave insufficient time for scientific analysis; and communications and Earth-based observations of the comet become more difficult because of the effective conjunction as the comet passes behind the Sun. Thus, science considerations most likely favor early arrival. A preliminary investigation examined the tradeoff between early arrival and mass delivery capability. The minimum acceptable mass performance based on a worst-case analysis excluded missions intercepting the comet earlier than 50 days prior to perihelion. This time was accepted as a mission constraint, although no definitive statement from scientists qualifies it as totally acceptable.

Late arrival, to the extent allowed by science, thermal, and communication constraints, becomes a contingency option and increases the mass delivery performance or increases the range for acceptable values for hardware performance tolerances.

2. Rendezvous condition. The second element in the definition of mission success is the rendezvous condition. The selection of a 50-day pre-perihelion arrival point determines the position elements of the final state vector at the comet. For purposes of this sensitivity study, an acceptable rendezvous is taken as that which results in zero relative velocity

between the comet and the vehicle. This occurs when the SEP thrust subsystem reduces the hyperbolic approach speed  $V_{HP}$  to zero. Minimum performance capability is associated with the  $V_{HP} = 0$  state. Relaxation of the relative velocity to a slow flyby condition again represents a contingency. The definition of the lowest acceptable relative velocity depends on the availability of authoritative science objectives and understanding of instrument and navigation capabilities. In the interim, the exact rendezvous,  $V_{HP} = 0$ , is defined as the acceptable mission boundary for hardware specifications.

3. Launch period. A third major element in defining acceptable mission boundaries is the launch period. Although the approach was to obtain basic data which is independent of launch vehicle specifications, it is also necessary in developing hardware tolerances to consider launch vehicle and mission operational requirements. Flight project plans for early launches to the outer planets nominally require a minimum of 21 days for dual-launch missions employing the Titan launch vehicle. This requirement can be relaxed slightly to a demand of 15 days for a single launch program. The actual period selected will ultimately reflect the confidence of project management in handling unforeseen problems in launch operations. To ensure compatibility with worst-case conditions and dual-launch programs, the mission boundary was conservatively set at 30 days.

4. Thrusting periods. Finally, a qualitative objective was set for thrusting periods. The lack of sophisticated mission simulation programs precluded detailed analysis to determine the maximum acceptable coast periods. However, preliminary work showed that the best mass capability was achieved over the most probable range of hardware operation on trajectories having little or no coast periods. In present studies, coast periods are optimally placed by the computer to maximize performance. The study guidelines eliminated definitive statements regarding coast trajectories. However, it was recognized that mission boundaries used in selecting hardware limits should include, to the extent allowable, the minimum acceptable conditions. For this reason, some consideration of coast periods is necessary because, realistically, there should be a "reasonable" amount of coast during the flight. Therefore, the hardware tolerance results include undesignated coast periods. A summary of mission boundaries is given in Table 1.

## V. ANALYSIS

### A. Important Parameter Combinations

Because of the close interaction of hardware and trajectory over a long period of time, more parameters must be analyzed for a low-thrust than for a ballistic trajectory. Data must be carefully handled so that the displays show the relationships consistent with the approach discussed in Section IV.

The most important parameter which relates thrust-subsystem hardware technology development to performance is the final mass  $m_f$  (the space vehicle mass at arrival). Using final mass as a trajectory success criteria facilitates reallocation of mass between the thrust subsystem and the other subsystems, including the science payload, without a priori knowledge of power subsystem specific mass.

The force which delivers the final mass is embodied in the kinetic energy contained in the thrust exhaust beam. Beam power is the effective power remaining after all the elemental losses have occurred. In the equations of motion, beam power enters as the combination of the elemental losses and the input power  $P(r)$  through the relationship

$$P_j(r) = \eta_{TSS} P(r) \quad (6)$$

Equation (6) is the instantaneous value of the beam power resulting from the instantaneous values of  $\eta_{TSS}$  and  $P(r)$ . The objective of the study was to determine the minimum acceptable value of the collective parameter  $\eta_{TSS}$ .

The value of  $\eta_{TSS}$  could change during the trajectory. Because the system may operate at nominal efficiency over only part of the trajectory, the design limit on efficiency must be based on the entire trajectory. Thus, the conditions for setting the minimum acceptable value were selected as those at the poorest anticipated operating point. If the design is based on such minimum performance limits, then nominal or superior performance on any part of the trajectory will increase the probability of success.

Mass and beam power occur as a ratio in the equation for thrust acceleration, Eq. (3). For determination of each trajectory, this equation

is integrated between initial and final values, after substitution in the equation of motion, (4). If the other control parameters in Eq. (4), specific impulse and pointing vector ( $\underline{u}$ ), are given, then a spectrum of trajectories can be represented by the associated ratios of final mass to initial beam power where beam power includes the value of efficiency ( $\eta_{TSS}$ ) at  $t = 0$  for a given input power ( $P$ ) at  $t = 0$ . With this ratio as the objective function, the values required for setting hardware boundaries can be combined and displayed for each trajectory. The use of this ratio as the objective function, instead of mass alone does not affect the trajectory optimization in the range of beam powers considered in this study (Ref. 2).

A similar parameter combination was used to account for propellant requirements. The instantaneous mass appearing in the vehicle acceleration term of Eq. (4) includes both hardware and propellant. The objective function ratio  $m_f/P_j$  is related to the mass at any time by

$$\frac{m}{P_j} = \frac{m_f + m_p}{P_j} \quad (7)$$

Thus, the propellant specific mass with respect to reference beam power  $m_p/P_j$  is an integral part of the trajectory calculation; and the value at  $t = 0$  determines the allowable initial mass for each trajectory and thrust subsystem beam-power combination.

The other control considered was  $I_s$ . The study objective included determination of an acceptable design value, which was accomplished by treating it as a parameter in the calculations so that its effects on the objective ratio  $m_f/P_j$  could be determined.

As mentioned previously, no control was imposed through the thrust pointing history. The trajectory computations allowed the thrust vector to follow any pointing history which maximized the objective function. Similarly, although coasting trajectories are required, no control was set on placement or duration of the coast periods.

#### B. Available Contingencies

Setting limits or specifications for thrust subsystem design and operations requires understanding all available mission contingencies or controls, such as arrival time, launch period, and coast periods. For arrival times

and launch periods, contingency is added to a system meeting the success boundaries, if the mission boundary definition is altered to allow later arrival times and shorter launch periods. Decisions about the contingency effect of coast periods are dependent upon further study.

Other contingencies, not considered as mission success criteria, are important as controls indirectly affecting mission success. In general, the ability to change the controls, which define the low-thrust mission mode, is available during three pretarget phases: (1) the initial design, (2) post-hardware delivery, and (3) post-launch. The number of controls available for respecification diminishes with each phase. Table 2 summarizes the controls available during each phase, including those discussed in Section IV-A. Criteria have been set for launch excess capability and launch period in Section IV-B. To a lesser extent, the arrival date is also set insofar as vehicle design mass requirements can be anticipated. The vehicle design mass is considered to be an outside input for this study and is based on preliminary configuration studies. The solar power reserve can be considered as part of the assumed 18% array-degradation. If the degradation is not as severe as anticipated, the reserve or excess power is available to the thrusters. Specification of the propellant reserve is intimately connected with the coast period design and must be such that launch can be made within the 30-day launch period and such that additional thrusting can be provided, if coast periods are shortened. The control, thrust pointing history, can be tailored during the design phase for the expected reference path. If a fairly narrow constraint is imposed because of look-angle requirements of other subsystems, then the adjustment flexibility in the post-hardware delivery and the post-launch phases is limited.

Table 2 shows that during the initial design and construction phase limits can be set and tradeoffs can be made among mission and hardware parameters to define mission success and set hardware specifications. Once the hardware which meets those specifications is delivered, adjustments can still be made should late considerations demand redefinition of mission goals. After launch, however, thrust subsystem anomalies can only be handled by adjusting the planned coasting periods, accepting later arrival at the target, using the planned solar power reserve, and altering the path with a new thrust-pointing profile.

1. Adjustment of coast periods. As discussed previously, launch energy can be treated independently of the launch vehicle for the mission under consideration. The low-thrust trajectories of interest are determined by the behavior of the initial acceleration  $a_0$  as a function of launch excess speed  $V_{HL}$ . For any combination of  $a_0$  and  $V_{HL}$ , launch excess capability may exist for a given launch vehicle in that it can deliver more mass at the selected launch excess speed or more speed for a given mass. The SEP module, sized in the design phase, will be expected to have a predicted mass and beam power. If no limits are imposed on these characteristics, the actual module delivered may be more massive than anticipated, while the available beam power could be exactly as expected. The result, as seen in Eq. (3), would be the reduction in the actual  $a_0$  achievable by the thrust subsystem. Without some adjustment of the trajectory, the resulting final state would not be the desired one. If the launch vehicle selected has the additional capability, the first alternative might be to raise the amount of launch energy supplied to compensate for the lower-than-planned contribution of the SEP module. For the Encke rendezvous mission, this alternative is available only over a very limited range. Figure 1 shows that if SEP module initial acceleration goes below about 0.36, increases in  $V_{HL}$  have no effect in saving mission success. Thus,  $V_{HL}$  has limited value for electric systems and has low threshold values of initial acceleration for this mission.

The contingency in reduction of the required launch period has a similar behavior. Figure 3 shows the behavior for several combinations of thrust subsystems and propellant specific masses, defined in relation to initial beam power. Final acceleration is plotted on the abscissa introducing the reciprocal of the objective function discussed in Section V-A ( $m_f/P_j$ ). The plot indicates the sensitivity of the required launch period to acceleration reductions. A delivered hardware system which is heavier than expected or which has a substandard beam power, can be compensated for, within limits, by reducing the 30-day launch period. For example, contingency for post-hardware delivery adjustments for a thrust subsystem with a specific propellant load of 49 kg/kW and  $I_s = 3,000$  s can be included by specifying a delivered hardware mass  $m_f$  and beam power  $P_j$ , which results in a calculated  $a_f$  of  $5.44 \times 10^{-4} \text{ m/s}^2$ . The lowest value which could be accepted without violation of the 30-day constraint would be  $5.36 \times 10^{-4} \text{ m/s}^2$ .



As seen in Fig. 2, the best performance returns are realized for flight times greater than 950 days. Shorter times drastically reduce performance, whereas longer times provide only modest gains. Figure 4 illustrates the mission contingency available by allowing arrival nearer perihelion. If the actual final mass is greater than expected, or the delivered beam power less than expected, the ratio of the mass and beam power which must be delivered,  $m_f/P_j$ , is increased. Such an increase may make arrival at 50 days before perihelion impossible. However, the figure shows that contingency is available for increases in  $m_f/P_j$  if the acceptable mission boundary is redefined, and, furthermore, that it is possible to readjust the propellant load specific mass ratio  $m_p/P_j$  to the appropriate value to maintain the 50-day point. This readjustment is required to impart the necessary energy increase through additional thrust time. The result of this adjustment is the probable decrease in the amount of coast time available for in-flight contingency.

As noted previously, the use of coast time as a control for setting hardware specifications was treated in this study in a qualitative manner only because of the non-availability of an appropriate simulation program. The approach used, illustrated in Fig. 5, typifies the basic data used for the operational analysis of the hardware sensitivity (Section V-C). The figure is based on the 50-day arrival time and illustrates a delivered thrust subsystem with an  $I_s$  of 3,000 s. The auxiliary power allowance is given as a ratio which includes the thrust subsystem efficiency  $\eta_{TSS}$ . The solid lines represent various values of the objective function  $m_f/P_j$ . Each point is a possible trajectory for the vehicle with that specific  $m_f/P_j$ . The path flown depends on the launch date. All the displayed trajectories include some amount of coast, except those lying on the dotted line, which denotes the continuous thrust boundary. The paths farthest to the left of this boundary have the largest amount of coast. Allowance for use of planned coasts as contingency is accomplished by constraining the allowed launch dates with the second dotted line denoted "launch period closed." This line is arbitrarily placed to provide a reasonable allowance of coast time and to reserve available paths for in-flight contingency use (post-launch phase). The propellant load ratio must be based on using this contingency. For example, if the actual delivered hardware has an  $m_f/P_j$  of 125 kg/kW, a propellant

reserve ratio, which theoretically allows thrusting through the planned coasts and up to the boundary, is  $m_p/P_j \approx 47 \text{ kg/kW}$ . Selection of this value would include some contingency for in-flight performance loss. The rationale for use would be as follows: Suppose launch occurs on March 1, with  $m_f/P_j = 125 \text{ kg/kW}$  and a propellant load ratio of  $m_p/P_j = 47 \text{ kg/kW}$ . These values are based on a 30-day launch period, plus reserve. All the available contingency paths lie on the vertical line shown in Fig. 5. A failure reducing the in-flight  $P_j$  would instantly increase the required objective function  $m_f/P_j$ . Concurrently, the propellant load ratio  $m_p/P_j$  would increase. The result would place the spacecraft at a contingency trajectory point on the vertical line. The mission would still be possible provided that the new pointing history can be met and the new value of  $m_p/P_j$  is consistent with the  $m_f/P_j$ , i.e., sufficient propellant reserve is carried. As shown, the new trajectory is much closer to the continuous thrust boundary, and the coast contingency is reduced.

2. Using the solar power reserve. Solar power reserve was not investigated in this study. The large uncertainty in the amount of solar array degradation, which can be expected, makes meaningful analysis difficult at present. It was recognized, however, that the profile for  $P(r)$  in Eq. (4) determines  $P_j$  and therefore the number of paths available to the hardware system. As a result, a conservative approach must be taken; and the additional power which may be available, if the degradation or auxiliary power requirement is not as expected, cannot be relied on in setting preflight development specifications. Even so, a requirement less than expected will provide contingency for in-flight adjustments even though an a priori quantitative prediction appears unlikely.

3. New thrust-pointing profile. Thrust-pointing capability and its interaction with planned contingency coast periods remains as an important and relatively unstudied control. Only with the availability of detailed targeting simulation programs can the capability to alter the mission through a changed pointing program and/or coast profile be evaluated.

### C. Hardware Sensitivity Analysis

The objective of this portion of the study was to examine the sensitivity of mission performance to the combined subsystem parameters,  $\eta_{TSS}$  and  $I_s$ , as defined in Eqs. (1) and (2) and thereby derive the constraints which should be imposed on the thrust subsystem hardware delivered. The approach taken was (1) to sequentially examine each of the contingency factors available and their impact on necessary thrust subsystem performance, starting from an assumed nominal mission and spacecraft preliminary design, (2) to examine the effect of variations of  $\eta_{TSS}$  and  $I_s$  on these contingencies and the constraint boundaries for subsystem performance, and (3) to determine the effect of design changes on these boundaries.

To implement this approach, the status of the spacecraft after launch was examined. The vehicle then has a fixed propellant mass, a fixed dry mass, a fixed solar-array area, a fixed thruster array, and a fixed beam voltage. The variables still available are: time-of-arrival, coast arc lengths, thrust-pointing history, and possible additional power available from the reserve allocation for solar array degradation. Of these, only thrust-pointing history is considered a free variable. To take advantage of any of the others, some contingency planning must have been previously incorporated, such as inclusion of sufficient reserve in the propellant load to permit thrusting during designed coast arcs to compensate for lower-than-nominal thrust subsystem performance.

The nominal space vehicle considered has the parameters given in Table 3. In the event of subnormal thrust subsystem performance, the contingency path(s) selected will depend upon the type of off-nominal behavior experienced. The curves shown in Fig. 6 depict the ratio of the maximum allowable final mass to initial jet-power ratio based on the selected mission success boundaries and the corresponding ratio of required propellant mass to initial jet power as functions of  $I_s$ . For the assumed nominal space vehicle, the actual values are  $m_f/P_j = 122.5 \text{ kg/kW}$  and  $m_p/P_j = 47 \text{ kg/kW}_j$ , which are well within the constraint boundaries.

The effect of finding, after launch, that the thrust subsystem performance was less than anticipated was then investigated. As a starting point, nominal values were assigned for  $\eta_2 = 0$ ,  $\eta_1 = 0.9$ , and  $\cos \theta = \cos \alpha = \cos \zeta = 1$  in Eqs. (1) and (2). The effect on  $\eta_{TSS}$  and  $I_s$  was plotted as these

parameters varied from nominal. One such case is shown in Fig. 7, where, for  $\eta_1 + 2\eta_2$  held constant at 0.9, the effect of  $\eta_2 \neq 0$  is shown for the cases of  $\bar{\theta} = 0$  and  $\bar{\theta} = 10$  deg. (It is assumed that beam voltage and, hence, power efficiency are fixed.) The data from Fig. 7 was then used to determine "actual" values of  $m_f/P_j$  as a function of  $I_{sp}$  (Fig. 8). By superimposing the curves of Fig. 6, it can be seen that the constraint boundary is violated for  $I_s$  values less than 2910 s; i. e., the thrust subsystem performance is too low to deliver the 1261-kg spacecraft to the destination, even with the continuous thrusting. More significant, even at 2910 s, 536 kg of propellant are required to deliver the 1261-kg spacecraft, i. e., 56 kg more than the nominal amount. To provide this reserve propellant, the planned coast periods must be reduced; i. e., if the subsystem performed nominally, it would still require thrusting through a substantial portion of the planned coast periods in the nominal missions. The exact effect of this was not calculated because of the limitations of the trajectory program used. The limiting case was taken as the 2910-s point, which in turn sets limits on the allowable variations of  $\eta_2$  and  $\bar{\theta}$  from their nominal values of zero, as shown in Fig. 9, wherein  $\eta_2$  is plotted as a function of  $\bar{\theta}$ . For  $\bar{\theta} = 0$ , the maximum allowable value of  $\eta_2$  is 0.040, decreasing to 0.028 for  $\bar{\theta} = 10$  deg.

The contingencies available after the delivery of the hardware, but prior to launch, were considered next. For this case, the solar-array area, beam voltage, spacecraft dry mass, and thruster array size are fixed, but propellant loading is still an available variable. If thrust subsystem performance is off-nominal and is discovered at this point, then propellant loading can be changed to accommodate the lower performance, assuming adequate tankage. The additional variable here, as opposed to the preceding case, is the launch date. If, however, the launch period cannot be violated, i. e., a launch window of less than 30 days is not acceptable, then the two cases are identical, and no relaxation of the above-mentioned constraints is possible. The significance of this case is that it can be used to redefine the propellant load and the nominal mission in a controlled manner.

The situation changes significantly if possible variations are considered during the preliminary design phase, when such fixed hardware parameters as beam voltage and solar array area can still be varied. Because the range of power levels of interest is far below the optimum value in terms of

mass delivery capability for the launch vehicles being considered, significant increases in propellant reserves and mass-delivery capability can be obtained by increasing the power level. This is illustrated in Fig. 10, wherein constraint boundaries on  $\eta_{TSS}$  at several values of  $I_s$  have been plotted for various power levels. These curves indicate that spacecraft dry masses of 1261, 1281, and 1301 kg, respectively, can be delivered for 16-, 17-, and 18-kW initial power to the thrust subsystem. These mission boundaries inherently include a given  $I_s$  versus  $\eta_{TSS}$  relationship. They must be updated for inclusion of variable  $I_s$  systems. Also shown is a band which covers the nominal subsystem performance over its expected operating range. The band accounts for efficiency and  $I_s$  variations with power level. It can be deduced from this figure that, as long as the path of the thrust subsystem operation from the nominal point A to some other point B does not cross the appropriate mission success boundary, then success, as measured by the delivered final mass for the selected power level, will be achieved. Such a path could result from throttling, etc. If, however, the path crosses the boundary, as typically shown at C, then mission failure occurs.

The figure shows that the operational range and, consequently, the interaction with mission success boundaries are strongly influenced by  $\bar{\theta}$  and  $\eta_2$ . For example, suppose point C is reached by some throttling function which maintains  $\bar{\theta}$  and  $\eta_2$  at zero. Several possibilities are then added, which can translate C as shown. If constant  $I_s$  is maintained, the dotted path results. This path reduces the effects of  $\bar{\theta}$  and  $\eta_2$ , showing that a system of 17 kW and  $M_f = 1281$  kg is still successful at full throttling with  $\bar{\theta} = 10$  deg and  $\eta_2 = 0.04$ . However, if constant  $I_s$  is not maintained, the same values of  $\bar{\theta}$  and  $\eta_2$  result in mission failure for the 17-kW system. From this data, it can be seen that by initially designing for 18 kW, substantial variations in the various parameters can be tolerated within the corresponding mission-success boundary. Further, if the true values of  $\bar{\theta}$  and  $\eta_2$  are known, the dotted line shows that the design power level can be reduced to 17 kW; and the corresponding success boundary is not violated, provided that the beam voltage is increased to maintain constant  $I_s$ .

The conclusion to be drawn from this is the importance of knowing, at the time of preliminary design, the exact values of such parameters as  $\eta_2$

and  $\bar{\theta}$ . Unplanned values for these parameters can, however, be accommodated by increasing the design-power level. Since this directly affects cost, the cost of minimizing allowable variances in subsystem parameters must be traded off against the cost of the additional power required to accommodate them.

A direct quantitative cost analysis was beyond the scope of this investigation. However, off-nominal subsystem performance during the various phases of the program has certain qualitative effects on cost. If off-nominal performance is detected during the preliminary design phase, or if provision for worst-case performance is made, the only cost increase is for the additional power required, which amounts to a few hundred thousand dollars. If such performance is detected after hardware delivery and it is necessary to change either the power level, the power conditioner, and/or the propellant loading, as well as flight software and mission operations, the cost goes up by an order of magnitude. If it is not detected until after launch and the mission constraint boundaries are violated, the result is mission failure, which costs on the order of one hundred million dollars. Therefore, the most cost-effective approach is to take the most pessimistic performance values, based on available data, for the mission design. Because the degree of pessimism depends upon the quality of data available, the real tradeoff is between the cost of reducing pessimism by better calibration and the cost of increased power for overly pessimistic assumptions. This tradeoff remains to be performed.

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2. Barber, T. A., Horsewood, J., and Meissinger, H., "Basic Parameters for Low Thrust Mission and System Analysis," AIAA Paper No. 72-426 presented at the 9th Electric Propulsion Conference, Bethesda, Md., April 17-19, 1972.

Table 1. Summary of mission boundaries

Parameter	Boundary
Arrival	50 days prior to comet perihelion
Velocity	Rendezvous at the comet ( $V_{HP} = 0$ )
Launch operations	30-day opportunity
Coast periods	A "reasonable" amount of coast time
Thrust vector pointing	No limitations placed on thrust pointing history for this study

Table 2. Available contingencies and controls

Controls	Initial design phase	Post-hardware delivery	Post-launch
Launch excess capability	X	X	
Launch period	X	X	
Arrival date	X	X	X
Coast periods	X	X	X
Vehicle design mass (defueled)	X		
Solar power reserve	X	X	X
Propellant mass (reserve)	X	X	
Thrust pointing capability	X	X	X



Table 3. Space vehicle parameters

Parameter	Nominal value
Dry mass	1261 kg
Propellant load	480 kg
Power to thrust subsystem	16 kW at 1 AU
Thrust subsystem $I_s$	3000 s
Thrust subsystem efficiency	65% (at full power)

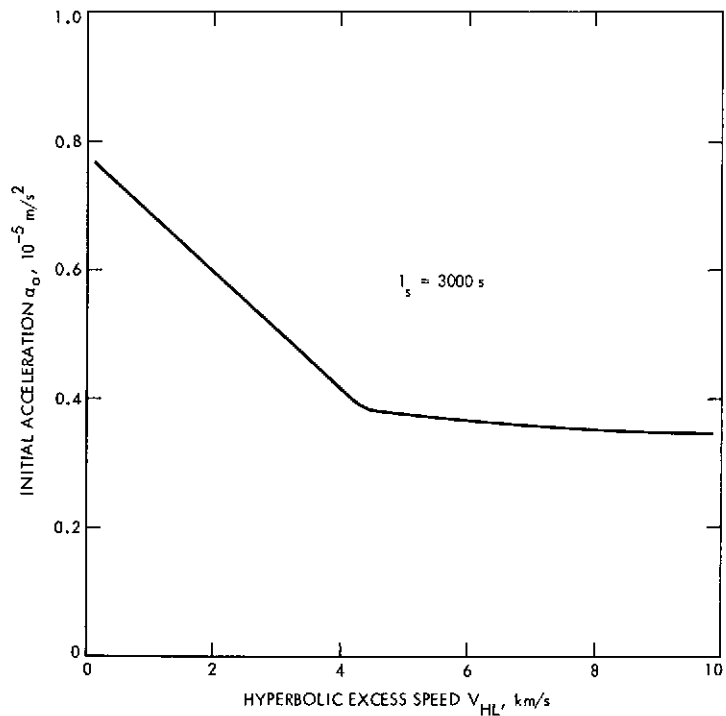


Fig. 1. Launch speed, acceleration, and tradeoffs for 950-day Encke rendezvous mission

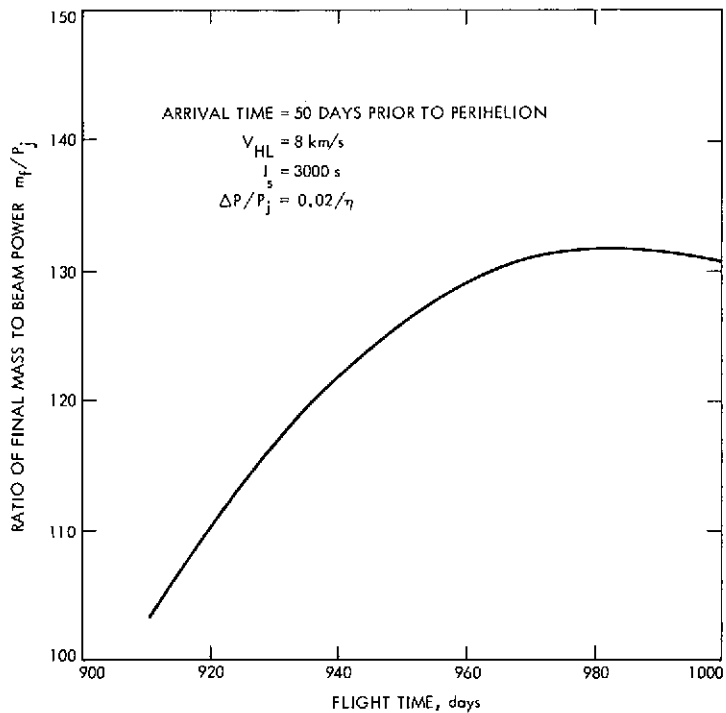


Fig 2. Data for selection of flight time, 1980 Encke rendezvous mission

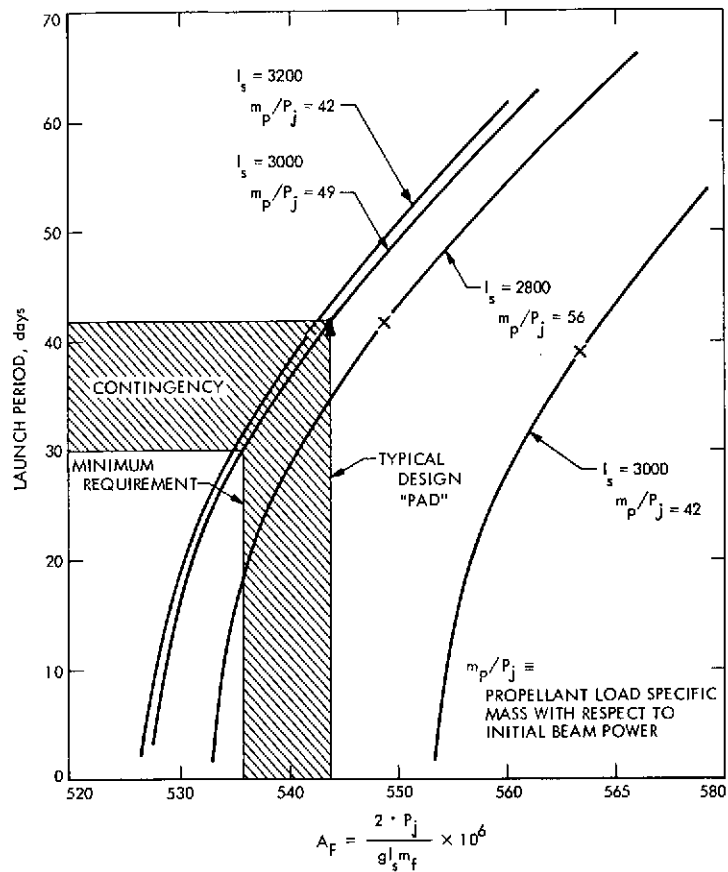


Fig. 3. Example of mission contingency in launch-period adjustment

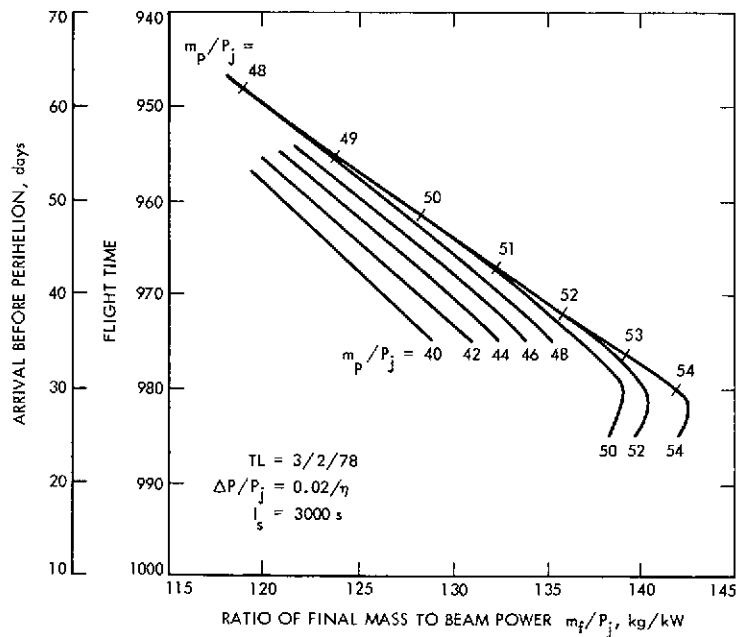


Fig. 4. Mission contingencies in arrival and propellant adjustments

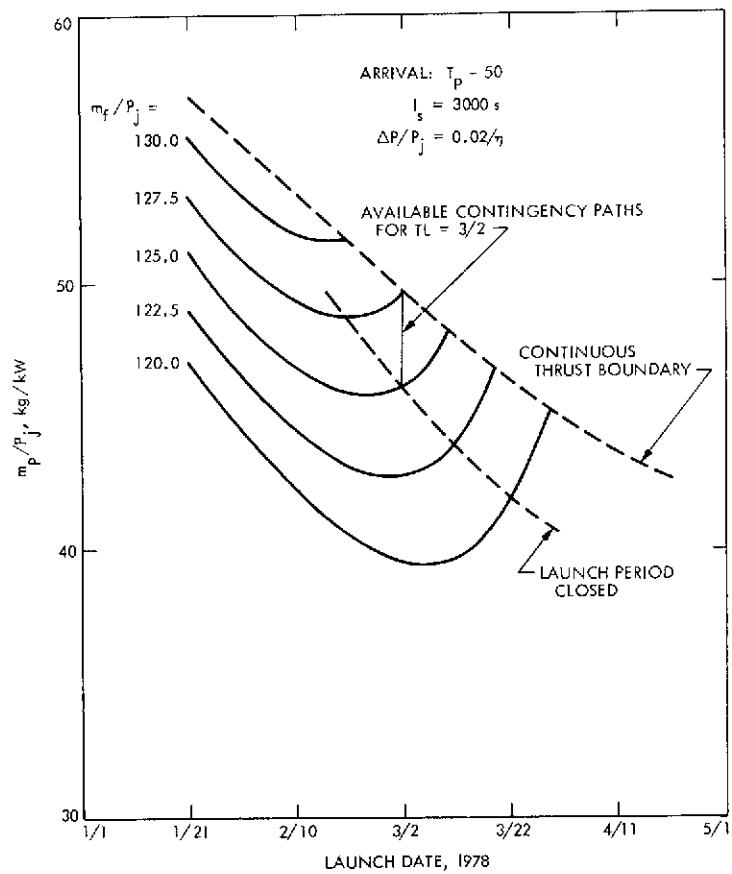


Fig. 5. Mission contingencies available in launch periods

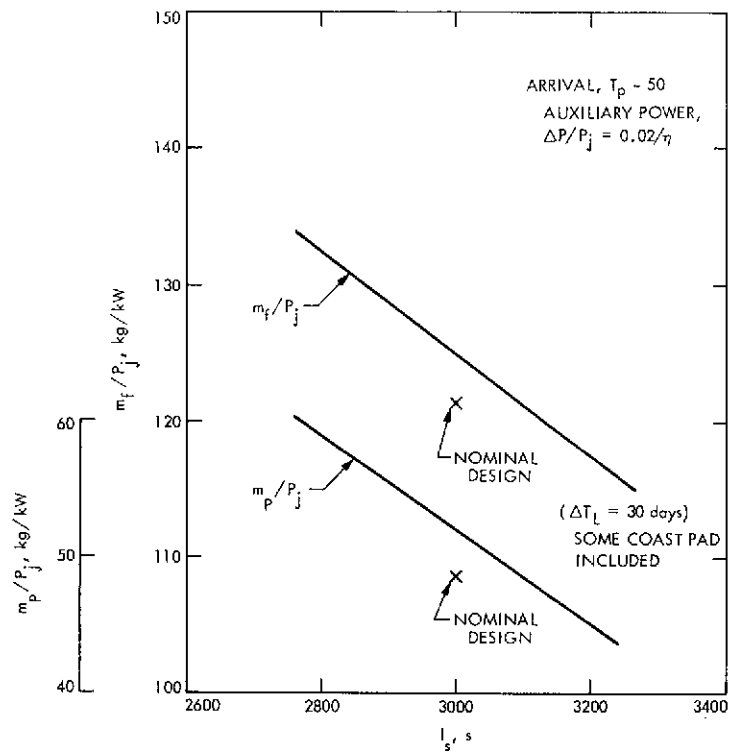


Fig. 6. Mass and propellant ratios  
 as functions of  $I_s$

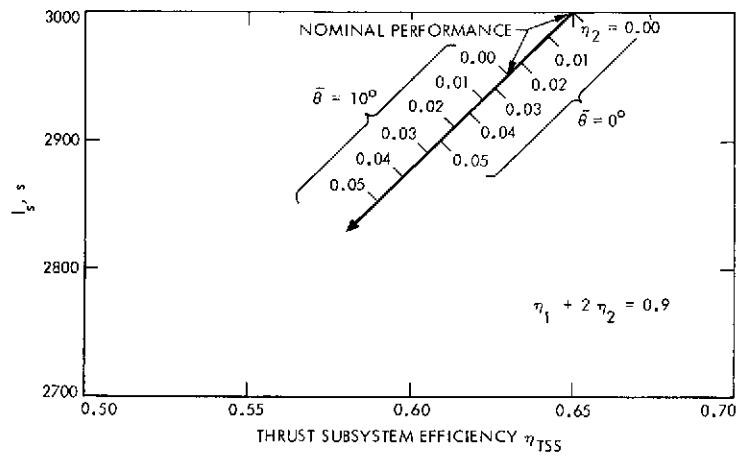


Fig. 7. Effect of off-nominal double ion content for 0- and 10-deg beam divergence angles  $\theta$

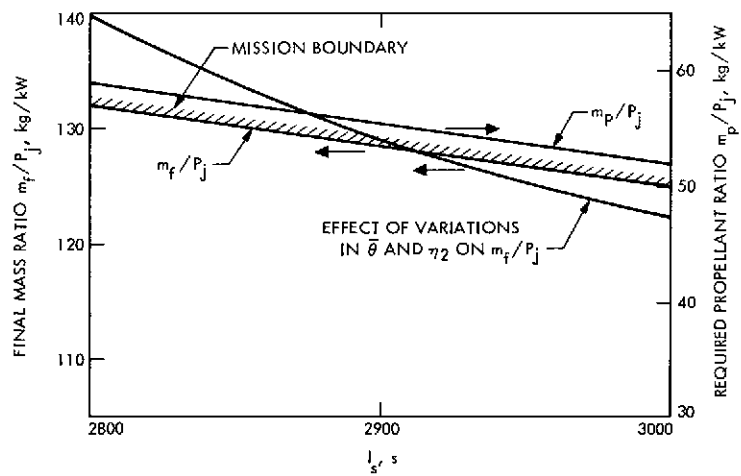


Fig. 8. Mission constraint on subsystem parameters

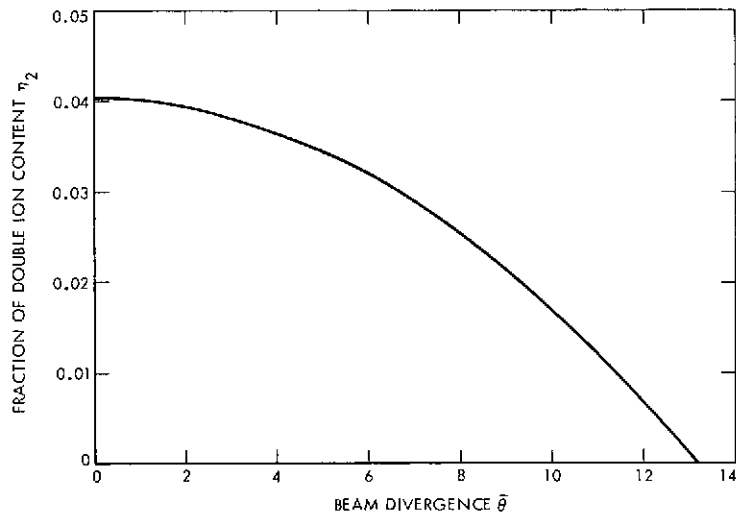


Fig. 9. Limit combinations of double ion content and beam divergence angles

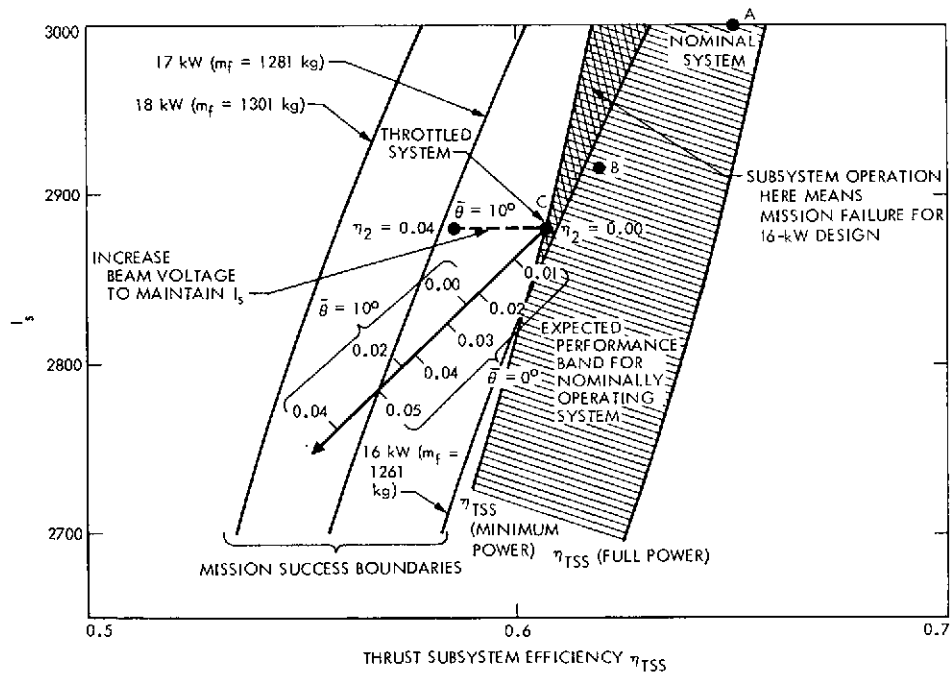


Fig. 10. Constraint boundaries and nominal expected performance for thrust subsystem efficiency  $\eta_{TSS}$

## PART 2. SENSITIVITY ANALYSIS

### I. INTRODUCTION

In the many years of analyzing low-thrust missions, in particular solar-electric-propulsion missions, a consistent and satisfactory method of performing a mission design analysis has been lacking. In many of these studies, a design point has been chosen that allowed for little if any variation in off-nominal vehicle or flight parameters. Since there is such an intimate relationship between the vehicle parameters and the trajectory variables such as launch date, flight time, and departure and arrival energy, the effects of one cannot be entirely separated from the others and all must be considered in a realistic mission design analysis. This report attempts to present a reasonable, although perhaps not complete, mission analysis philosophy to be followed together with examples for an Encke comet mission for illustration.

### II. MISSION PARAMETERS

In order to perform a comprehensive mission analysis, a number of spacecraft and trajectory parameters must be considered. It is also desirable that the trajectory data be generated independent of the launch vehicle characteristics and independent of spacecraft parameters such as propulsion system specific mass and thruster efficiency in order to make the results of the analysis as general as possible. Thus trajectory data should be generated using spacecraft thruster specific impulse and initial thrust acceleration or spacecraft specific mass  $M_0 / P_J$  as free parameters.

In addition to these spacecraft parameters, there are a number of trajectory variables which must be considered for a particular mission although some of these may be constrained for the mission being investigated. These trajectory parameters include launch date and arrival date or flight time and also departure and arrival energy. Note that if the entire



unrestricted range of possible trajectory and spacecraft parameters is to be investigated, an inordinate number of trajectories would need to be calculated and data stored. This number of free parameters can be reduced somewhat by constraining the arrival date and arrival energy for some missions, but trajectories would still need to be generated for possible ranges of launch energy and launch date and for the desired values of thruster specific impulse. In addition, sufficient values of initial thrust acceleration or spacecraft specific mass will be required in order to adequately describe mission performance.

When a general knowledge of the mission and launch vehicle is available, it is possible to restrict the range of launch energies that need to be investigated. For many missions such as asteroid or comet rendezvous or slow flyby where a low (rather than optimum) powered SEP spacecraft is assumed, the performance will be found to be relatively insensitive to launch energy and only one or two values of launch energy need be investigated in a preliminary mission analysis. In outer planet missions, the performance is more sensitive to launch energy and a more detailed mission design may be necessary.

In the analysis which follows, values of propellant specific mass  $M_p/P_J$  were determined as functions of both spacecraft initial specific mass  $M_0/P_J$  and spacecraft final specific mass  $M_f/P_J$  for various values of thruster specific impulse, launch energy, launch date and arrival date or flight time. Note that the effective thruster jet power  $P_J$  is used rather than thruster input power in order to divorce the analysis from specifying, a priori, thruster characteristics. Of the various forms of presenting normalized mission analysis data, the use of specific mass appears the easiest to comprehend for the individual not familiar with mission analysis terminology. In addition, spacecraft specific mass is easily compared with propulsion system specific mass, which is a spacecraft design parameter and not a mission parameter.

It is not the intent of this report to present an exhaustive number of mission design plots but rather to present typical presentations of data which may be found useful to the user and which will indicate the potentialities of the analysis presented in this report. It should be noted that much of the detail given in this report is not essential for every mission design, and a

certain amount of data editing, described in a later section, can be done by the computer to reduce the quantity of data required for the analysis.

An example of the data generated for a particular mission, where the launch date, flight time, arrival energy, and thruster specific impulse were fixed, is shown in Fig. 1. In this figure values of propellant specific mass are shown as a function of final spacecraft specific mass for various values of launch energy (given in terms of launch hyperbolic velocity  $V_{HL}$ ). The data for this mission, that of an Encke slow flyby, is typical of the data for SEP asteroid or comet missions. Note that the maximum values of propellant and final specific masses occur at a point corresponding to the continuous thrust limit. This limit represents the constraint beyond which it is not possible to accomplish the mission with the specified constraints. Only by allowing additional freedom in flight time, arrival date, or arrival energy might it be possible to change the regime over which it is possible to operate.

Again from Fig. 1, note that it is desirable to operate with as high a value of final spacecraft specific mass as possible since, for a fixed final (dry) mass, the jet power and hence spacecraft propulsion system mass is minimized. It might appear that going to as high a value of launch energy as possible would be preferred, since for the values shown in Fig. 1, this results in the highest values of final specific mass. Note, however, that there is a launch vehicle performance constraint inherent in any mission analysis, and the operating regime (i. e., launch energy) must be chosen so that this constraint is not violated.

The data from Fig. 1 was used together with the launch performance of a Titan IIIE/Centaur booster to generate the curves shown in Fig. 2. These curves are no longer normalized since the data has been combined with the launch vehicle capability. This data is presented in a form more commonly seen in which final spacecraft mass is shown as a function of jet power for the same values of launch energy as in Fig. 1. Again the continuous thrust limit is shown together with curves of spacecraft initial specific mass  $M_0/P_J$ . The values of specific mass can be used together with values of final mass and jet power to calculate the propellant mass for any point falling within the operating regime. Alternately, curves of constant propellant mass could also have been shown in Fig. 2, although these were not calculated for this example.

Note in Fig. 2 that the performance is relatively insensitive to launch energy for values of excess velocity above 6 km/s. Thus nearly the same values of specific mass result when using these launch energies. This introduces the concept of spacecraft and propulsion system "scaling," where the full launch vehicle capability is not employed but rather the spacecraft is scaled in size along a line of constant  $M_F/P_J$ .

As an example, using the full launch vehicle capability at  $V_{HL} = 7$  km/s, it is not possible to perform this mission with a jet power of 10 kW since this point falls to the left of the continuous thrust line (Fig. 2). However, by following a line of constant  $M_F/P_J$  of 126.8 kg/kW, representing nearly the maximum value for this launch energy, it is possible to perform the mission with a final mass of 1268 kg, a propellant mass of 492 kg, corresponding to a propellant specific mass of 49.2 kg/kW from Fig. 1, and an initial mass of 1760 kg. Since the launch vehicle capability at this energy is 2195 kg, a launch performance margin of 435 kg results.

If instead the full launch vehicle capability was used at a greater launch energy of  $V_{HL} = 7.8$  km/s, a final mass of about 1280 kg would have resulted which used 490 kg of propellant and required an initial mass of 1770 kg. Note that a net gain of 12 kg in final spacecraft mass results from using the full launch vehicle capability as compared to a launch vehicle reserve of 435 kg at an excess velocity of 7 km/s.

In performing a mission analysis, quite frequently it is the final mass which is known as a function of jet power, assuming certain parameters of the propulsion system. Thus by providing data such as in Fig. 2, the spacecraft designer can determine an operating point by using this data together with the variation of final mass with jet power.

Such an example is shown in Fig. 3, where an example of the variation of final mass vs jet power has been included. In this example, a spacecraft final mass as a function of jet power of

$$M_F = 600 + 50P_J$$

is assumed in order to determine a mission design point. Note that the mission can be performed with a jet power anywhere to the right of the

continuous thrust curve shown in Fig. 3. As higher values of jet power are used, operation farther from the continuous thrust limit occurs (i. e., longer coast arcs) and departures are at lower values of launch energy.

Assuming a value of 10 kW for the desired jet power and a final mass of 1100 kg results in a spacecraft final specific mass of 110 kg/kW. Employing the concept of scaling described previously and assuming values of launch excess velocity of 5, 6, 7 and 8 km/s, the values of spacecraft parameters shown in Table 1 resulted.

### III. VARIATIONS OVER LAUNCH DATES

Obviously a complete mission design must include more than the point design just described. Problems to be resolved include those of the effect of launch date variations, the effect of variations in arrival date, and the effects of variations in thruster specific impulse. For a preliminary mission design, generally the last two items mentioned above can be constrained and only variations in launch date examined.

By fixing specific impulse and arrival date, variations in launch date could be analyzed by generating data similar to that presented in Fig. 1 for each of the launch dates in question. If this data had to be generated for each of the values of launch energy considered for Fig. 1, a large amount of data would be required. Since the performance in many cases is relatively insensitive to launch energy for the missions being considered, it may only be necessary to examine one or two specific values of launch energy.

An example of an Encke rendezvous mission is used for the remainder of the examples presented in the sections following. In addition, since the effects upon performance of launch date are being examined, the launch date will be displayed on the abscissa of the plots shown in the following figures. The arrival date will also be fixed at 50 days before comet perihelion so that the flight time will vary over the range of launch dates examined.

In Figs. 4 and 5 the values of propellant specific mass are given as a function of launch date for fixed values of spacecraft specific mass. Figure 4 presents the performance in terms of fixed values of spacecraft initial specific mass; Fig. 5 presents that data in terms of fixed values of spacecraft final specific mass. Both figures utilize a value of launch excess velocity of

7 km/s. The points for these figures could have been determined by interpolating curves such as the one shown in Fig. 1. However, the values of propellant specific mass were calculated by an interpolation from a data bank generated previously for this mission. The details of the calculation of the interpolated values are not important in this present discussion but are discussed in a later section.

At this point it is assumed that the mission analyst has a good estimate of the values of spacecraft final specific mass and is interested in determining the propellant to be used to perform the particular mission based on covering a particular launch period. In this context the data presented in Fig. 5 is most useful since the user can first determine whether the mission is possible and, second, the desired range of launch dates that may be covered. Note that the best launch date varies to some extent with the value of final specific mass selected.

Some observations can be made regarding the data presented in Fig. 5. First, there exists a maximum value of final spacecraft specific mass of approximately 126 kg/kW at a flight time of 970 to 980 days. This point represents the optimum design point commonly given in describing maximum mission capability for the arrival date and specific impulse used in this example. The use of this design point allows for no additional contingencies in off-nominal hardware performance other than that assumed initially. Also the effects of a delay in launch date would necessarily result in arrival, for this mission, at a later time than the value assumed here of  $T_P - 50$  days. Thus in order to provide for mission contingencies of various kinds and also to allow for propulsion system design that is not overly constrained in regard to hardware tolerances, a design point should be chosen somewhat to the left of the continuous thrust limit to allow for some moderate coast phases during the nominal mission. A more detailed description of the factors entering into the selection of an operating point is given in Part 1 of this report and these selection criteria will not be elaborated upon further. The remainder of Part 2 of this report presents the method whereby the mission maps can be used to perform a mission sensitivity analysis and a description of the programs used in support of this analysis.

#### IV. USE OF MISSION MAPS FOR A SENSITIVITY ANALYSIS

The use of Figs. 4 and 5 in performing a mission sensitivity analysis will now be described. The discussion will be limited to the one mission set wherein rendezvous with Encke is at 50 days prior to perihelion. Note that an additional contingency option is to allow a free arrival date. In an actual mission this should be used as a contingency option to allow additional degrees of freedom for unexpected thruster failure modes (Ref. 1) and for the correction of probable errors during the terminal rendezvous guidance phase. For this analysis and the sensitivity study in Part 1, a fixed arrival date has been assumed, however.

In order to apply the results of this analysis to an example, some assumptions must be made regarding the spacecraft propulsion parameters. In the examples to be presented, the values listed in Table 2 were assumed for the nominal spacecraft design point. In this example, the combination of a desired final mass of 1200 kg and a jet power of 10 kW yields a spacecraft final specific mass of 120 kg/kW. In Fig. 5, the curve for a value of  $M_F/P_J = 120 \text{ kg/kW}$  is seen to fall well to the left of the continuous thrust limit and will result in an acceptable mission. Arbitrarily adopting a requirement for a 30-day launch period, a propellant specific mass of 46 kg/kW is indicated. This specific mass and a thruster jet power of 10 kW imply a propellant requirement of 460 kg and spacecraft initial mass of 1660 kg and a spacecraft initial specific mass of 166 kg/kW.

From Fig. 4, the curve of initial specific mass  $m_0/P_J = 166 \text{ kg/kW}$  is used to determine the amount of propellant actually required as a function of launch date. Since the propellant actually loaded represents a propellant specific mass of 46 kg/kW, it is apparent that launch is prohibited on dates requiring higher propellant specific masses than the above. From the figure, it is seen that a minimum propellant specific mass of 45.2 kg/kW, equivalent to a propellant mass of 452 kg, is required during the middle of the launch window. This minimum propellant mass of 452 kg implies that a residual propellant mass of 8 kg will remain at the end of the mission for a launch during the middle of the launch period. This 8 kg of unused propellant can be considered as an additional contingency option for missions launched near the middle of the launch period.

Note also from Figs. 4 and 5 that earlier launches are characterized by longer coast phases since they occur farther from the continuous thrust limit. Although not indicated, lines of constant coast time would lie approximately parallel to the continuous thrust limit curve. Since the end of the indicated launch period occurs quite close to the continuous thrust limit, it might be necessary to terminate the launch period some days earlier to allow a trajectory with some minimum amount of coasting.

## V. PARAMETER SENSITIVITY

As mentioned previously, the use of higher launch energies provides for a small contingency option. A mission map similar to that given in Fig. 4 is shown in Fig. 6 for a launch energy corresponding to an excess velocity of 8 km/s. An increase in performance of about 2.5 kg/kW is available by going to this higher launch energy. With the spacecraft parameters given in Table 2, a propellant load of approximately 420 kg is required for a final mass of 1200 kg. The resulting initial mass of 1620 kg slightly exceeds the assumed injection capability of the Titan IIIE/Centaur at this launch energy, however.

The curves given in Figs. 4 and 5 may also be used in a sensitivity analysis to observe the effects of variations in spacecraft propulsion parameters that do not involve changes in thruster specific impulse. Examples of possible contributors include variations in initial spacecraft mass, variations in thruster efficiency and variations in the reference solar array output power. In order to examine the effects of variations in thruster specific impulse, a separate set of mission maps must be created using this graphical technique.

An example of the effect of a variation in reference solar array power is presented in Table 3 and Fig. 7. Here the effect of an increase of 5 and 10% in solar array output power is shown. This particular variation is important since it indicates the sensitivity of performance to assumed values of solar array degradation that are used in preliminary mission performance estimates.

This example has been calculated assuming that the thrust system can handle the additional solar array output power. From Fig. 4 an increase in

performance (in terms of final mass) of 5 and 10% is possible by using the same design values as before ( $M_F/P_J = 120 \text{ kg/kW}$ ). This increase in performance corresponds to an increase in final mass capability of around 60 and 120 kg, respectively. If, on the other hand, the initial mass is constrained to a value of 1660 kg as in the example given in Table 2, the propellant requirements shown in Fig. 7 result. Note that the required propellant is less than that required for the example using the nominal power for a particular launch date even though the propellant mass flow rate is greater with the increased solar array power. That the propellant requirements are less reflects the presence of the longer coast phases which result using the increased thruster power, which more than compensates for the increased rate of usage of propellant.

If the propellant loading is kept the same, the effect is to increase the apparent launch period, or if the launch period is fixed, the effective residual propellant is increased over the launch period. Again, the extra propellant may be used as a contingency option to reduce flight time (i.e., earlier arrival) or to allow for additional propulsion capability after encounter. Note that the effect of the additional power available to the propulsion system results in operation further from the continuous thrust limit and hence in trajectories with longer coast phases.

In order to examine the effects of off-nominal specific impulse variations, additional mission maps would need to be generated. Figure 8 presents several curves (not the entire mission maps) for three values of specific impulse. The propellant mass requirements are given for a fixed initial mass of 1660 kg as in the nominal example. Variations of  $\pm 200 \text{ s}$  in specific impulse were considered, corresponding to a specific impulse of 2800 and 3200 s. A different value of thruster efficiency was assumed for each example of specific impulse, reflecting the change in thruster efficiency with specific impulse. The nominal propellant load is also indicated in Fig. 8 as the horizontal dashed line at a propellant loading of 460 kg.

Note from Fig. 8 that the propellant required corresponding to a 2800-s specific impulse is greater than for the nominal value of 3000 s and results in a very short launch period of 5 to 10 days. If it is known, a priori, that the specific impulse is low, additional propellant could be loaded so as



to increase the launch period. Operation is further from the continuous thrust limit for low values of specific impulse, and the coast phases will be longer than for the nominal example.

Although it might be expected that a higher value of specific impulse would result in a wider launch period, such does not occur since the continuous thrust limit has moved to an earlier launch date and has the effect of terminating the launch period prematurely. Note from Fig. 8 that the operation at lower values of specific impulse is preferred (at least for this example), particularly so if the value of specific impulse is known before liftoff and additional propellant loaded.

## VI. THE USE OF GRAPHIC TECHNIQUES

The preceding sections have discussed the use of graphical techniques in performing a preliminary mission design and performance sensitivity analysis. While the methods and presentation of data are generally straightforward, nevertheless the generation of the many mission maps is time-consuming, even when performed by computer techniques. Furthermore, the mission maps are difficult to interpret if values must be interpolated between curves.

With these considerations in mind, the following multiple-step approach is taken. First, parametric data is generated over the expected range of flight times, launch dates, and launch and arrival energies for the mission being considered. This "raw" data is stored in a data file by the computer or alternately output on punched cards for more permanent retention. This data is considered normalized or parametric in the sense that it does not include a postulated launch vehicle, power, or thruster parameters. The only spacecraft parameter entering into this data is the relative solar array power profile. The parametric data is then used with the following iterative procedure to determine an acceptable mission set of spacecraft parameters and to analyze the effects of parameters variations on the mission:

- (1) A nominal set of mission parameters, i.e., launch energy, flight time, and arrival energy, is chosen to be examined.

- (2) A nominal set of hardware parameters is selected, including thruster input power, specific impulse, efficiency, and desired final spacecraft mass.
- (3) With the use of either an available set of mission maps or the postprocessor program to be described later, the particular set of mission parameters is investigated to determine its suitability. If the selected set of hardware parameters is not suitable (e.g., if the mission is not possible), another mission set is selected or another set of hardware parameters is investigated.
- (4) An acceptable set of parameters having been found, the propellant mass is determined as a function of launch date for the value of final spacecraft mass selected.
- (5) A propellant load is determined based upon covering a specified launch period and including additional propellant reserves if desired. The desired launch period should be specified beforehand as an input quantity, and would typically be around 20-30 days.
- (6) An initial spacecraft mass is determined based upon the desired propellant loading, and the launch vehicle injection capability is checked to determine if it is sufficient at the specified launch energy. If the injection capability is insufficient, either the spacecraft parameters must be changed or a lower launch energy employed.
- (7) The initial mass and propellant loading having been specified, the actual propellant required to perform the mission is determined as a function of launch date. Based upon the amount of propellant loaded, the actual launch period is determined and the propellant reserve calculated as a function of launch date.
- (8) With an acceptable nominal mission set and hardware set of parameters determined, the effects of off-nominal performance can be investigated.

## VII. DESCRIPTION OF CHEBYTOP

Previously, no mention of computer programs used to generate this mission analysis data has been made. Almost any low-thrust mission analysis program can be used to generate the data for the mission maps presented previously. However, a low-thrust mission analysis program called CHEBYTOP developed by The Boeing Company (Ref. 2) possesses some features that make it particularly attractive in analyzing the type of problem being considered herein.

A detailed description of CHEBYTOP may be found in Ref. 2. However, in essence, CHEBYTOP is a two-part program that solves the unconstrained-thrust, power-limited, low-thrust trajectory by means of a series of patched Chebychev polynomials. The coefficients of these polynomials are selected so as to minimize the trajectory specific power and to satisfy the particular trajectory constraints. These unconstrained thrust trajectories are propulsion system and launch vehicle independent and only depend upon the initial and final state, vector flight time, and launch and arrival energy.

CHEBYTOP generates constrained thrust (constant specific impulse) trajectories by a scheme which uses the unconstrained thrust acceleration profile as a basis for approximating the multiple coast constrained thrust performance. The accuracy of the constrained thrust solution depends upon the paths of the unconstrained and constrained thrust trajectories being close to one another. For a preliminary mission design the accuracy in calculating performance for the constrained thrust solution is generally better than 3% as compared with a more exact integrated trajectory. The solution of the constrained thrust trajectory requires the input of various spacecraft parameters such as thruster specific impulse, thruster efficiency, thruster input power, and spacecraft initial mass.

The separation in CHEBYTOP between the unconstrained and constrained thrust solutions suggests that it should be possible to save or store the unconstrained thrust data which is required to generate the constrained thrust solution and perform our analysis at a later time using this data exclusively. This method appears quite attractive since by far the greatest amount of computer time is spent in generating the unconstrained thrust trajectory.

Initially, a set of unconstrained thrust trajectories is computed which covers the range of launch and arrival dates and energies to be considered. A separate data file is saved from each trajectory and stored in a data file by the computer or punched on cards for more permanent retention. Since CHEBYTOP has been written as a group of subroutines, it is necessary only to make a call to the unconstrained thrust subroutine followed by one call to the constrained thrust subroutine to generate the data to be saved. The data being saved contains several variables identifying the particular trajectory and an array of the unconstrained thrust acceleration magnitude computed at equal values of regularized time. If the propulsion time is desired, an additional data array which relates actual time to regularized time would also need to be saved. This data array is not now saved. However, it would be a simple matter to store it in addition to the acceleration data.

### VIII. DESCRIPTION OF POSTPROCESSOR PROGRAM

A breadboard program has been written identified by the acronym CMAP (Chebytop Mission Analysis Program) that processes the data previously stored from CHEBYTOP. CMAP makes use of a number of subroutines used for the CHEBYTOP constrained thrust solution and has been designed to accept data files from either version of CHEBYTOP, although at present the subroutines used in CMAP utilize only those from CHEBYTOP I because of several problems encountered when the constrained thrust system for CHEBYTOP II was used.

The CMAP program accepts as input either data files or punched cards. The program will read the data previously generated and store certain variables identifying the stored data in the computer memory and print out a tabulation of the launch dates, arrival dates or flight times, launch energy and arrival energy. The data records containing the acceleration history are stored in temporary Fortran data files in order to decrease the core storage or memory requirements for the program.

Values of launch and arrival energy are input together with either flight time or arrival date, depending upon the options selected. The particular data records for each launch date are retrieved from file storage and stored in memory. If the input data does not correspond to the identification data

stored in the tables, an error condition exists and the program returns for another set of input data.

In addition, values of initial or final mass, thruster specific impulse, efficiency, and thruster input power are also input. CMAP sets up different logic, depending upon whether the initial or final mass are input. The program will calculate the propellant required for each launch date being considered. If the given spacecraft parameters do not result in an acceptable mission for some of the launch dates, the launch dates are interpolated to determine that launch date that corresponds to a continuous thrust trajectory.

An initial propellant loading can also be included as an input quantity and the launch period corresponding to this propellant loading calculated. CMAP will indicate in this case whether the propellant load is sufficient for the example being considered, whether the launch period limits are outside the range of launch dates being considered, and whether the launch period terminates at the continuous thrust limit. CMAP will also calculate and print values of regularized (not real) propulsion time as a function of launch date.

The regularized time  $T_b$  and actual time  $T$  are related by

$$dT_b = \frac{P(R)}{P_0} dT$$

where  $P(R)$  represents the variation of solar array power with solar distance. Note that if a constant power (i.e., nuclear electric) case is being considered, the regularized time and actual time are the same. In order to calculate the actual propulsion time, an additional array relating regularized to actual time would be required. This was not considered necessary for the present study, however, since the estimates of true propulsion time can have relatively large errors as calculated by CHEBYTOP, even though the regularized propulsion time is calculated accurately.

A flow diagram which describes the operation of CMAP is shown in Fig. 9. The CHEBYTOP constrained thrust subroutines used in CMAP were modified slightly so as to be able to calculate the continuous thrust trajectories correctly. There are several possible problems inherent in both CHEBYTOP and CMAP. The acceleration profile saved from CHEBYTOP can be quite

irregular or noisy in certain cases because of the method of interpolating the data in CHEBYTOP. This irregularity results in convergence problems when going to the constrained thrust solution. The effects of the noisy data are manifested in many cases in multiple coast phases when only one should be present. A potential problem with CMAP occurs when the final mass is input, since the problem can be double-valued in some cases. This could occur since there may be two values of propellant mass that will result in the desired value of final mass. It has not been observed in the examples of the SEP Encke flyby or rendezvous missions, however, but has been observed in other examples, primarily constant power nuclear-electric missions.

#### IX. THE USE OF CMAP AS A MISSION ANALYSIS TOOL

This program has proved to be a valuable tool in analyzing the effects of parameter variations on mission performance and in performing an initial mission design analysis. The accuracy of CMAP is, of course, comparable with that of CHEBYTOP, and the types of mission constraints are limited to those that can be handled by CHEBYTOP. The most important requirement for CMAP, and also, incidently, CHEBYTOP, is for a smoother acceleration profile to be used for the constrained thrust solution.

## REFERENCES

1. Yen, C. L., and Smith, D. B., Low-Thrust Mission Risk Analysis with Application to a 1980 Rendezvous with the Comet Encke, Technical Memorandum 33-593, Jet Propulsion Laboratory, Pasadena, Calif., March 15, 1973.
2. Hahn, D. W., Johnson, F. T., and Itzen, B. F., Chebyshev Trajectory Optimization Program (CHEBYTOP) Final Report No. D2-121308-1, The Boeing Co., Seattle, Wash., July 1969.

Table 1. Performance variation with launch energy

$V_{HL}$ , km/s	$M_P/P_J$ (Fig. 1)	$M_P$ , kg	$M_0$ , kg	$\Delta M_{LV}$ , kg
5	43.0	430	1530	1918
6	37.0	370	1470	1352
7	32.8	328	1428	767
8	29.8	298	1398	193

$\Delta M_{LV}$  = excess launch vehicle capability.

$M_F = 1100$  kg;  $M_F/P_J = 110$  kg/kW.

Table 2. Encke rendezvous, 1980

Desired final mass	1200 kg
Thruster input power	16 kW
Spacecraft power	0.32 kW
Specific impulse	3000 s
Thruster efficiency	0.625
Thruster jet power	10 kW
Final mass/jet power	120 kg/kW
For 30-day launch window	
Propellant mass	460 kg
Initial mass	1660 kg
Minimum $M_P$	452 kg



Table 3. Effect of greater than nominal solar array power

$P_E$	16.8 kW	17.6 kW
$\Delta P_E / P_E$	5%	10%
Launch window	48 days	56 days
Propellant reserve	20 kg	35 kg

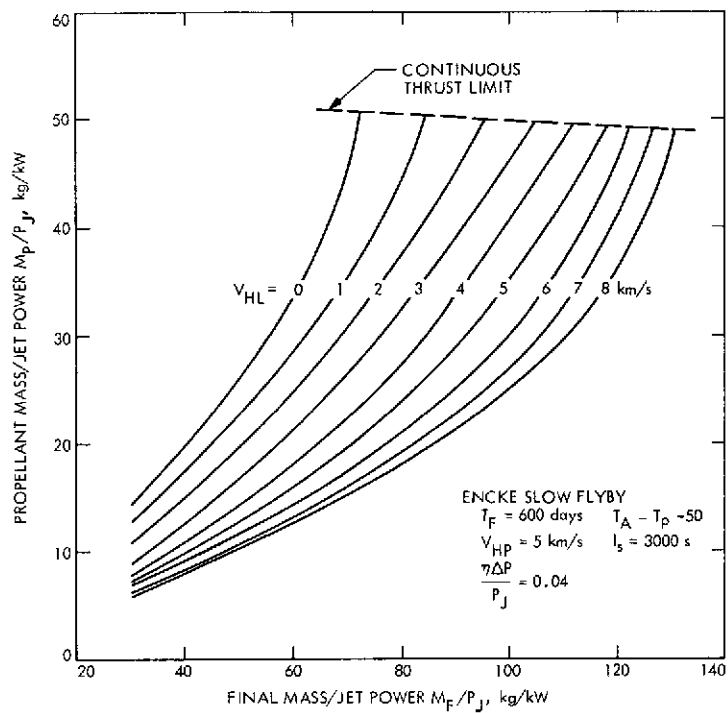


Fig. 1. Propellant specific mass vs final specific mass

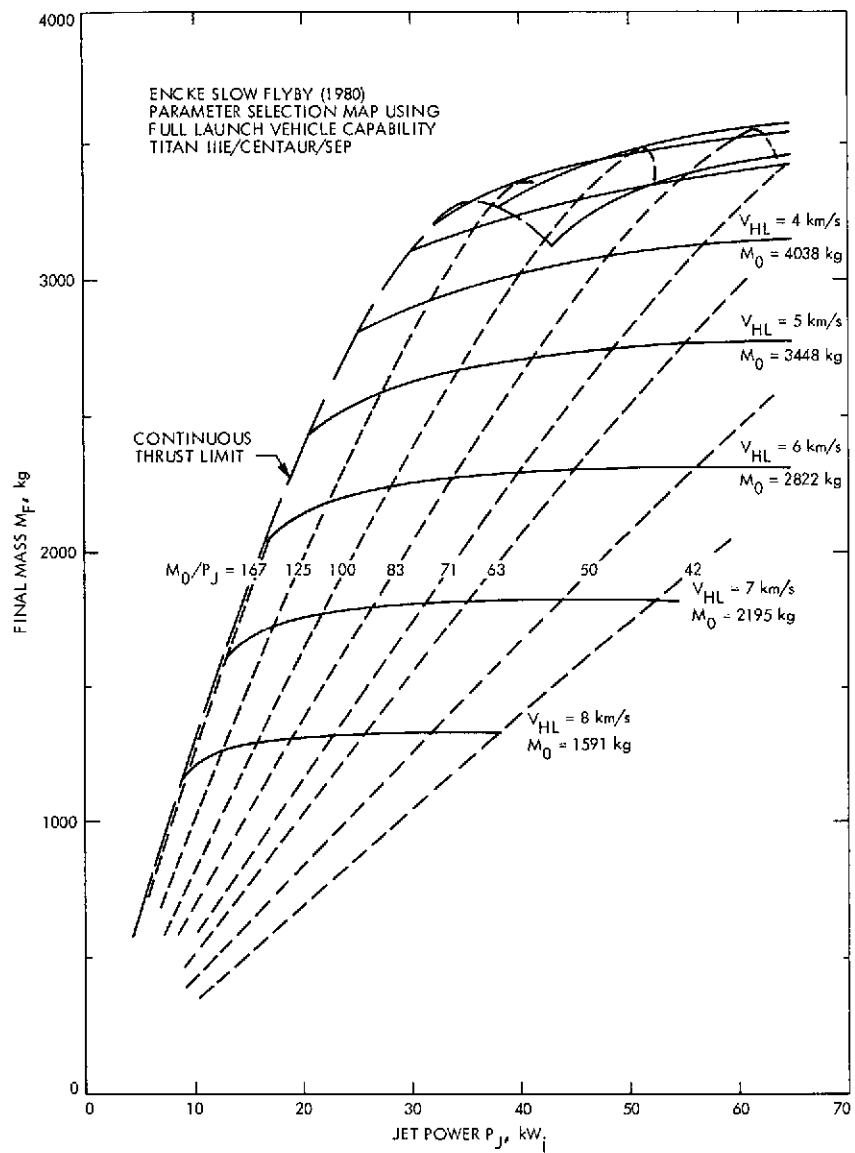


Fig. 2. Parameter selection map

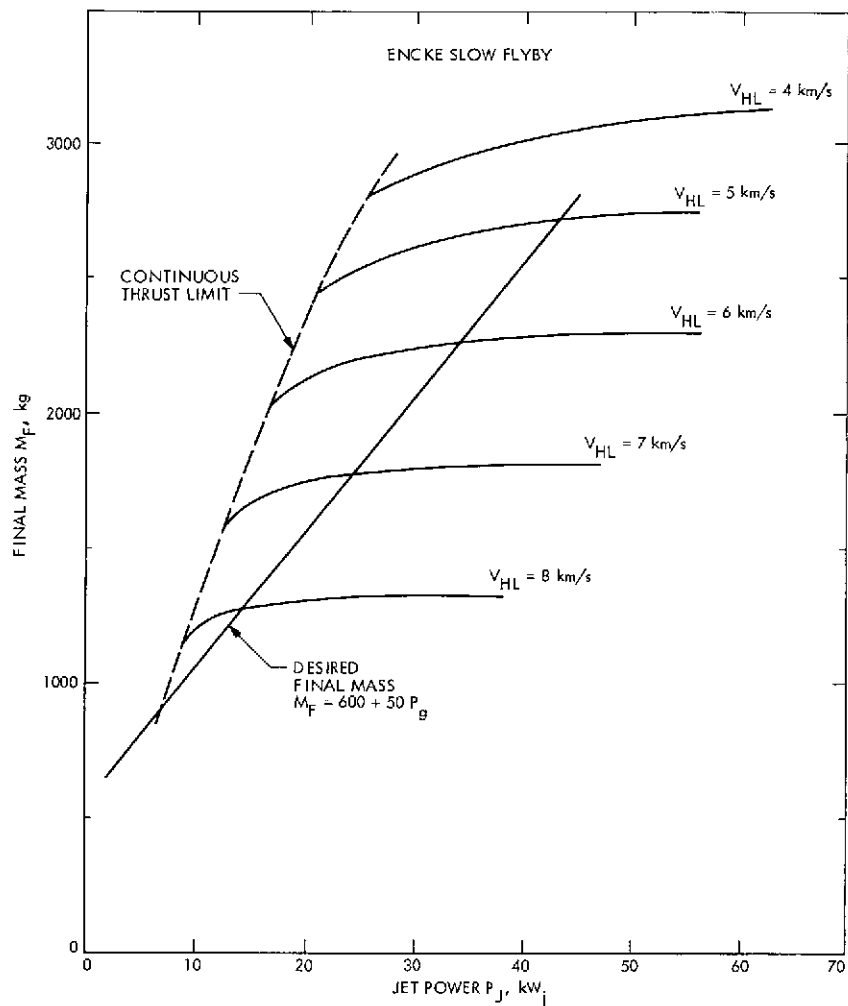


Fig. 3. Parameter selection map showing desired final mass

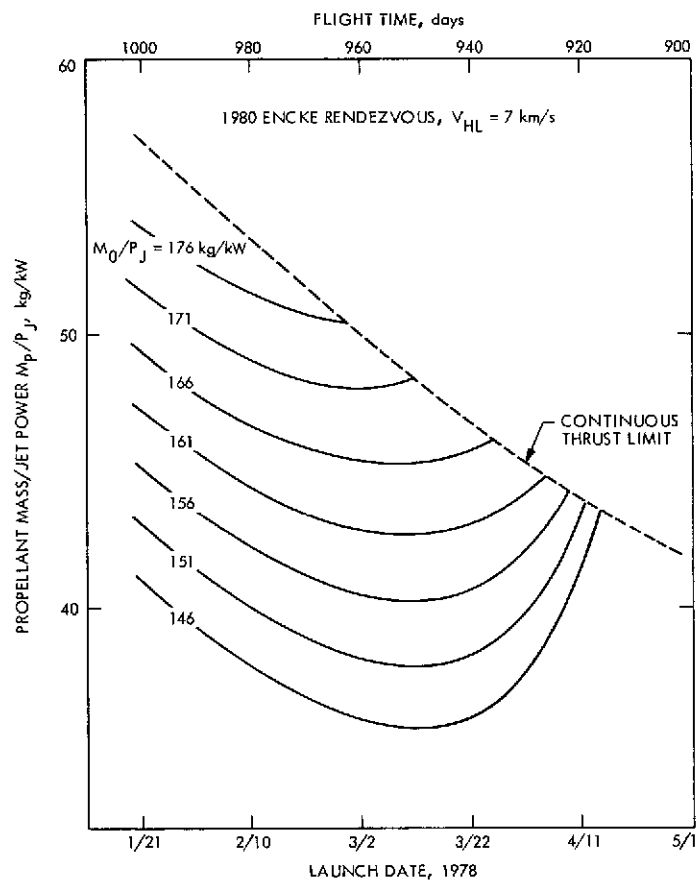


Fig. 4. Propellant specific mass vs launch date, initial specific mass as a parameter

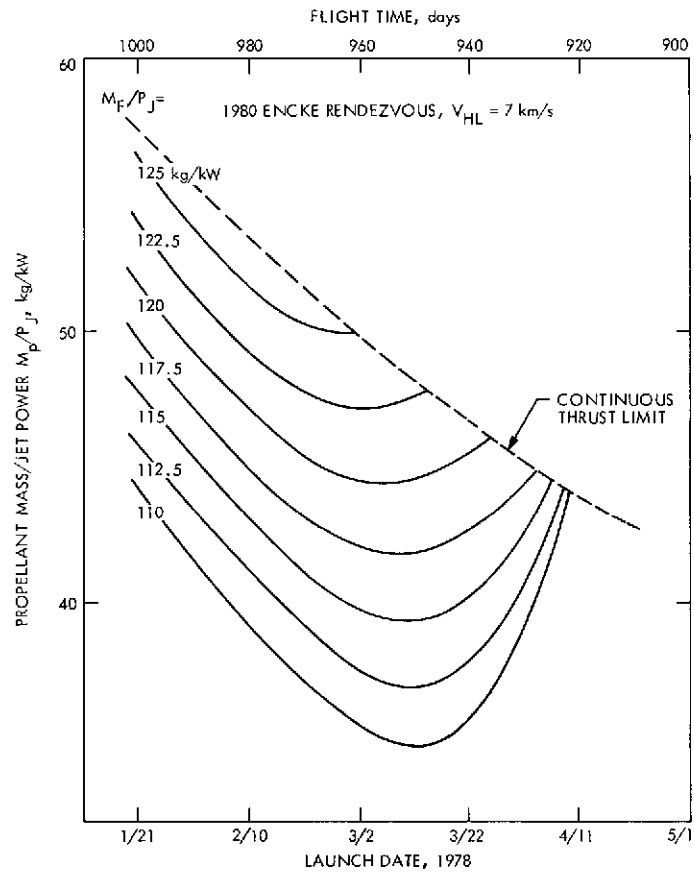


Fig. 5. Propellant specific mass vs launch date, final specific mass as a parameter

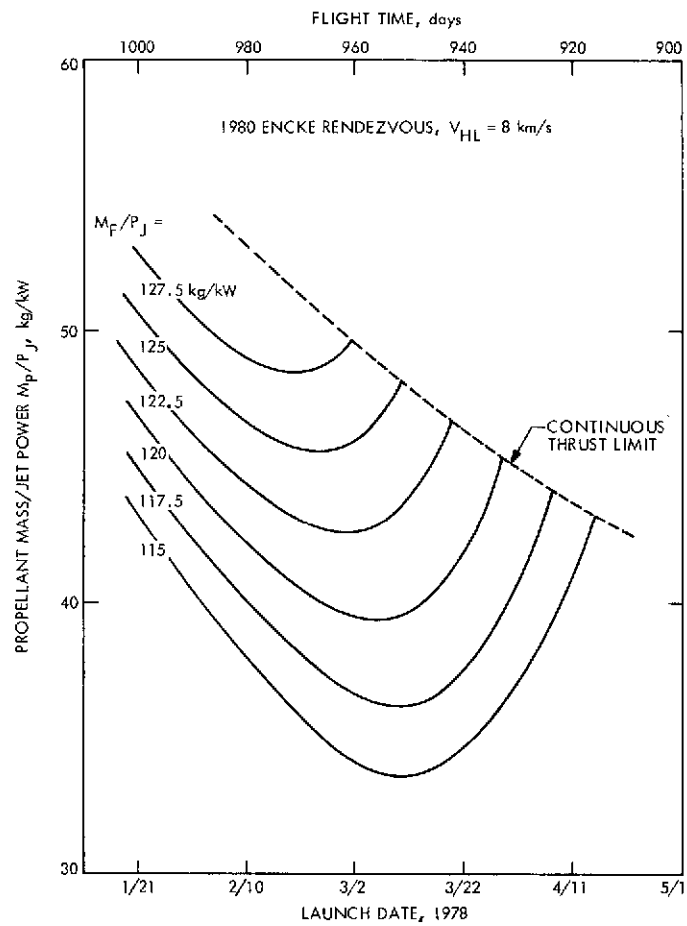


Fig. 6. Propellant specific mass vs launch date,  $V_{HL} = 8$  km/s

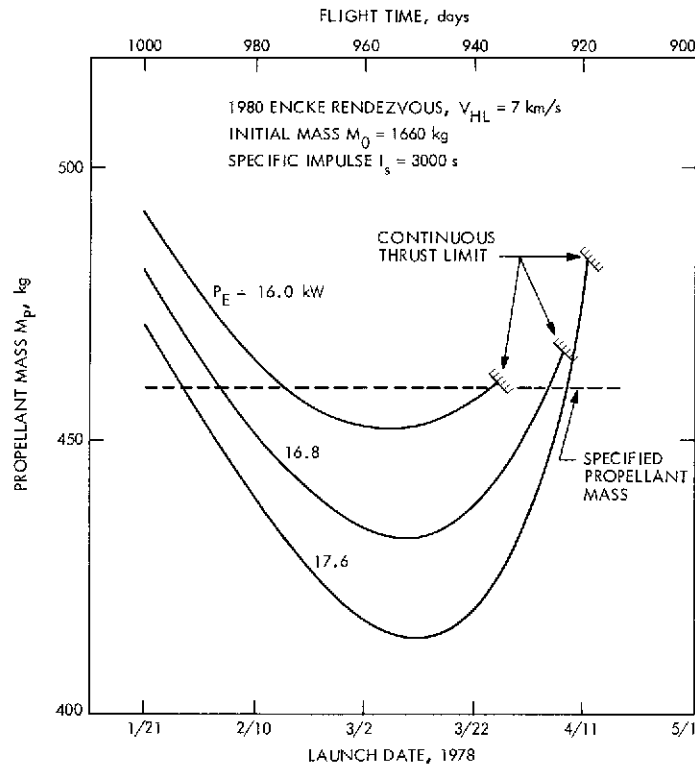


Fig. 7. Propellant mass vs launch date, variable power and fixed initial mass

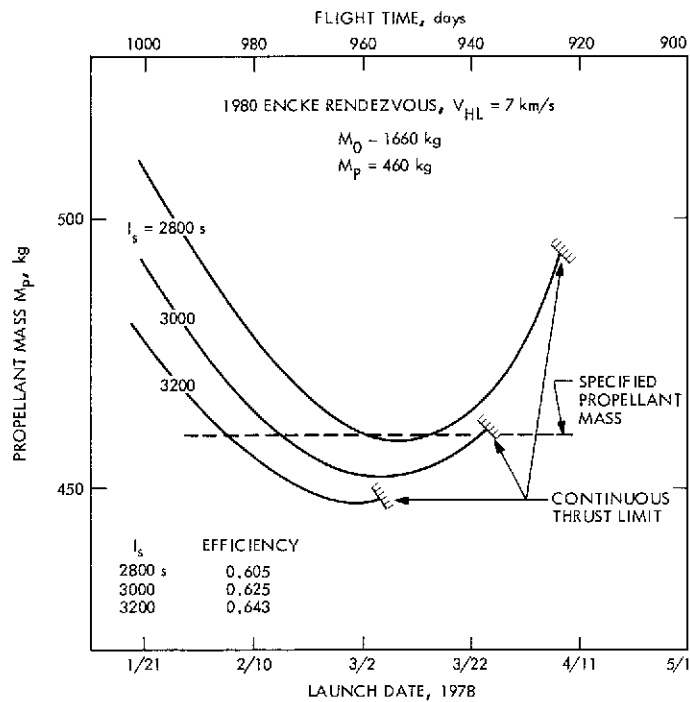


Fig. 8. Propellant mass vs launch date, variable specific impulse and fixed initial mass



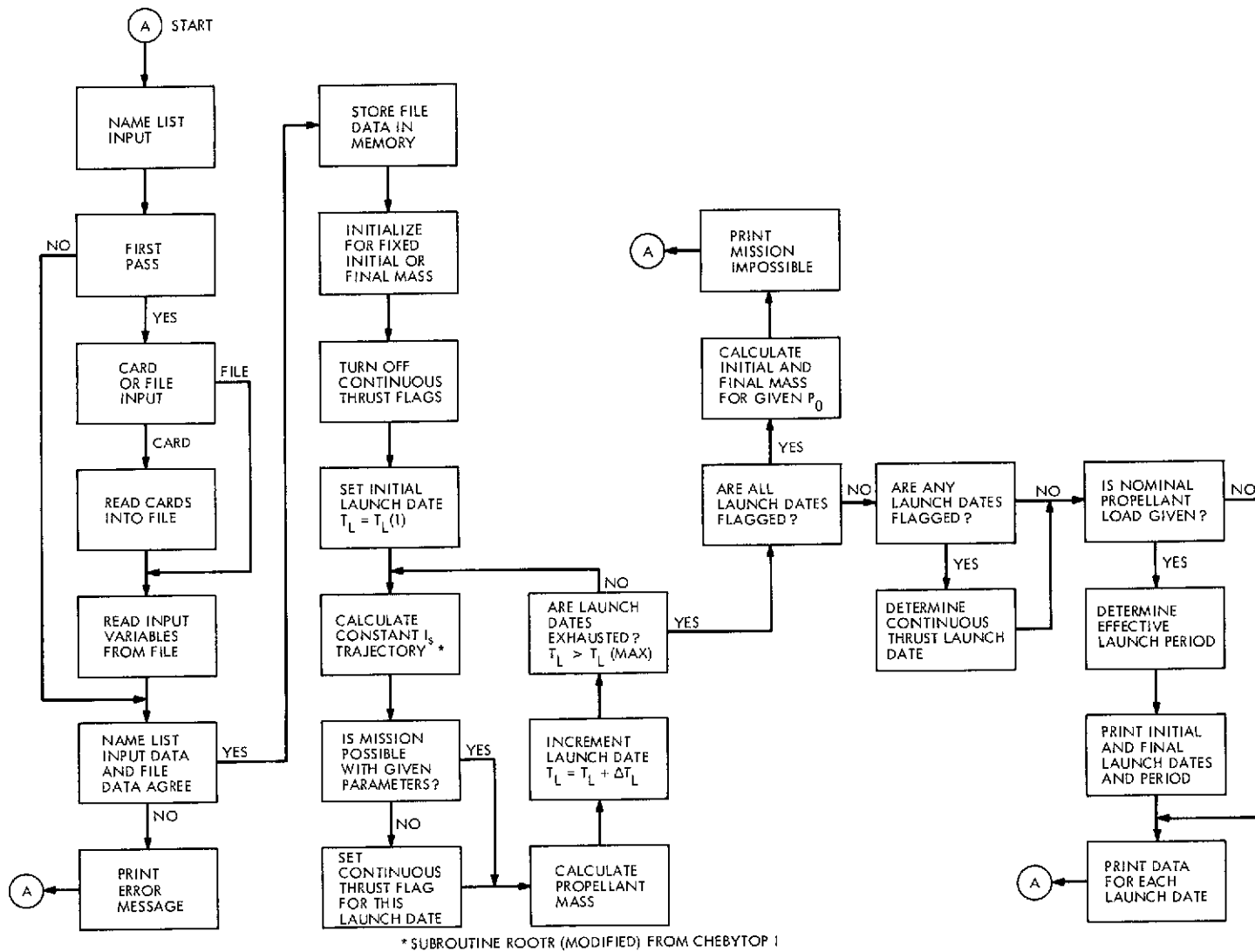


Fig. 9. Flow diagram of CMAP